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SCHEDULING CRITERIA FOR MULTIPROGRAMMING UNDER CERTAIN CONSTRAINTS IN A HARD REAL-TIME ENVIRONMENT

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SUMMARY. The problem of scheduling a set of tasks that must be regularly performed within certain intervals of time according to a given priority and sharing a unique resource, one at a time, was studied in [1] and the results published in [2]. In this note we study the feasibility of such scheduling in two particular cases, in which the mechanisms implementing the priority scheme are unable to observe it completely, failing to do so when all the tasks are simultaneously requested to be performed.

1. INTRODUCTION.

We consider a set of nodes that periodically request to transmit a message, sharing a unique resource, on the following conditions:

1 - The medium is granted to one node at a time and during a fixed interval of time, without interruptions. This time is the same for all nodes and it is called the slot-time.

2 - At the start of the process all nodes request simultaneously to transmit their messages.

3 - Each node request to transmit its message at regular intervals. The time elapsed between two consecutive requests of a node is called the crisis time of the node.

4 - The crisis times of the nodes are integer multiples of the slot-time.

5 - When a node requests to transmit, the corresponding message must be delivered exactly once before its next request.

6 - The start of any message is synchronous with the beginning of a slot.

7 - Each message can be transmitted within the slot-time.

8 - The medium remains idle only if none of the nodes requests it.

9 - The time unit is the slot-time.

According to the above assumptions, time is always measured by positive integer numbers. The time space (the slot space) is represented by the set of natural numbers and sometimes we shall say instant for slot.

If a, b are positive integers and $a \leqq b$, the interval [a,b] is the set of all integers such that $a \leqq x \leqq b$; if $a \nleqq b$ the interval [a,b] is the empty set.

When a message is not delivered before its deadline it is said that crisis occurs. The problem is to schedule the system avoiding crisis.

Following [2] we shall say that a priority discipline (PD) is given for a set of nodes if at the beginning of each slot a linear ordering relation is defined on the set of nodes. The ordered set is called the priority stack; at the beginning of each slot the medium is assigned to the first node in the stack among those with pending messages. Most of the deterministic (i.e. non random) priority disciplines suitable for real-time systems are variations or combinations of three main types of priority assignments: Round Robin, Fixed Priority and Least Time to Crisis.

In a Round Robin discipline (RRD) the priority stack for a given slot is obtained from the preceding one transferring the node at the top to the bottom. It is a dynamic discipline in the sense that the stack is a function of time.

In a Fixed priority discipline (FPD) the stack is assembled once for all; it is a static discipline. For instance, the stack could be ordered by increasing crisis times.

In a Least Time to Crisis discipline (LTCD) the stack at each slot is induced by the time available to each node before reaching crisis. If there are several nodes with the same time left these nodes are ordered according to a certain linear ordering chosen beforehand.

Given a set of nodes and chosen a priority discipline for it, we shall say that the system is crisis-free for that PD if none crisis occurs in the time interval $[1, +\infty)$. A crisis-free system will be said saturated if every slot is engaged in the transmission of a message. We shall say that a slot is full or empty according there is or there is not a message being transmitted at it.

Since at the start of the process all the nodes generate simultaneously their messages, this situation arises again for the first time at the instant M+1, being M the least common multiple of their crisis times. Therefore the system is periodic and to study its behaviour it suffices to analyse it in the time interval [1,M].

We summarize now some of the results published in [2], where it was always assumed that the mechanisms implementing the chosen priority disciplines are able to observe them to full extent. We shall need these results further on:

R1) A crisis-free system is saturated if and only if $\sum_{i=1}^{n} 1/T_i = 1$, being n the number of nodes and $T_1, \ldots, T_n$ their crisis times.

(If a system under a PD is crisis-free then $\sum_{i=1}^{n} 1/T_i \leq 1$, but the converse is not necessarily true.)

R2) ([2], Theorem 1). Given a crisis-free non saturated system operating under a PD, the i-th slot e_i left empty by the system is the least $x \in [1, +\infty)$ such that $\sum_{h=1}^{n} \lceil \frac{x}{T_h} \rceil = x - i$, $\forall$ $i \in N$.

($\lceil y \rceil$ denotes the smallest integer larger than or equal to y.)

Sometimes for simplicity we shall denote $f_{(n)}(x) = \sum_{h=1}^{n} \lceil \frac{x}{T_h} \rceil$.

R3) ([2], Theorem 5). Let $S(n) = \{\eta_1,\dots,\eta_n\}$ be a crisis-free non saturated system operating under a FPD and consider the system $S(n+1) = \{\eta_1,\dots,\eta_n,\eta_{n+1}\}$ obtained by adding a node and placing it at the bottom of the stack. Then S(n+1) is crisis-free if and only if $T_{n+1} \geqq e_1$, where e_1 is the first slot left empty by S(n) and T_{n+1} is the crisis time of η_{n+1}.

R4) ([2], Theorem 6). If S(n) is such that $n \leqq T_{min}$ then S(n) is crisis-free under any FPD, that is , no matter the chosen fixed stack. (T_{min} denotes the minimum crisis time in the system).

R5) From Theorems 1 and 7 in [2] it easily follows: If $S(n) = \{\eta_1,\dots,\eta_n\}$ is a system operating under a FPD and such that $n > T_{min}$ then S(n) is crisis-free if and only if it verifies

$$\text{for } i = T_{min}+1,\dots,n \qquad \sum_{h=1}^{i-1} 1/T_h < 1 \qquad \text{and} \qquad T_i \geqq e_{1(i-1)} , \qquad (1)$$

where $e_{1(i-1)}$ denotes the first slot left empty by the subsystem $S(i-1) = \{\eta_1,\dots,\eta_{i-1}\}$. (The nodes are numbered according to their decreasing priorities).

Condition (1) can be replaced by an equivalent (2), easier to be tested.

Using the characterization of $e_{1(i-1)}$ given by R2), it follows without much difficulty that (1) is equivalent to (2):

$$\text{for } i = T_{min}+1,\dots,n \quad \exists \quad x_i \in [i,T_i] \text{ such that } 1 + \sum_{h=1}^{i-1} \lceil \frac{x_i}{T_h} \rceil = x_i . \qquad (2)$$

2. TWO CASES OF DEFECTIVE OBSERVANCE OF THE CHOSEN PRIORITIES.

Now we shall consider a system with a Fixed Priority discipline operating under the following constraint: the mechanism implementing the chosen FPD serves it well, except when the worst case of load (simultaneous generation of messages at all nodes) takes place. At this instant the medium is granted to any node, not necessarily to the node with the highest priority, though from the next slot on the FPD is observed.

The problem is to characterize the systems that even working on these conditions do not reach crisis. It was proposed to us by Ing. Jorge Santos.

Since at the start of the process all the nodes generate simultaneously their messages and the system is periodic of period M, M the least common multiple of the crisis times, we may suppose that the failure of the mechanism in observing the FPD occurs at the first slot.

DEFINITION. A system operating under a Fixed Priority discipline will be said **steady crisis-free** if all the messages from the system can be transmitted without crisis provided the FPD is observed from the second slot on, no matter to what node is granted the first slot.

We shall say that the system is **crisis-free** if crisis does not occur while the FPD is fully observed from the first slot on.

A crisis-free system is not necessarily steady crisis-free, though the converse is obviously true.

We intend to give a necessary and sufficient condition for a system under a FPD to be steady crisis-free.

If S(n) is a system operating under a FPD, in what follows it will always be assumed that the nodes are numbered 1,...,n according to their decreasing priorities, and for simplicity they will be denoted 1,...,n; $T_1,...,T_n$ and T_{min} will denote their crisis times and the minimum among them.

We readily notice that

<u>THEOREM 1</u>. Any system S(n) such that $n \leq T_{min}$ is steady crisis-free whatever the FPD stack be.

Proof. If $n \leq T_{min}$ then S(n) is crisis-free no matter how the FPD stack is assembled by R4). In the interval $[1,T_{min}]$ only the first message of each node is generated. These messages can be transmitted within the interval and it is clear that the ordering of the transmissions does not matter. The occasional failure of the mechanism in serving the FPD at the first slot would not produce any further modification on the assignment of the medium from the $T_{min}+1$-th slot on. Therefore S(n) is steady crisis-free.

The above property means that we should restrict ourselves to the case $n > T_{min}$. Nevertheless we shall proceed to study a system S(n) without assuming such restriction on n. But it will be worthwhile to bear Theorem 1 in mind.

In order to establish the desired results it is convenient to study first another kind of defective observance of the FPD.

Consider the following situation: Given a crisis-free system operating under a FPD, suppose that the mechanism fails in granting the first slot and that it remains empty, that the node with the highest priority just gains the second slot and that from this slot on the FPD is observed. In this case we shall say that **the system skips.** If even so all the messages from the nodes can be transmitted without crisis, we shall say that **the system can skip.**

<u>DEFINITION</u>. Given a crisis-free system under a FPD, it will be said that the system can skip if all the messages can be transmitted without crisis even though the first slot is forbidden to the system and the FPD is observed from the second slot on.

2.1. A characterization of the systems that can skip.

We wish to give a necessary and sufficient condition for a crisis-free non saturated system to be able to skip. To this purpose we pay attention to the slots left empty by each subsystem S(i) consisting of the first i nodes, $i = 1,\dots,n-1$, and reckon that, as S(n) is crisis-free, the messages of each node i, $i > 1$, are fulfilled at slots left empty by S(i-1).

The effective computation of the empty slots is given by R2).

<u>THEOREM 2</u>. Let S(n) be a crisis-free non saturated system operating under a FPD. If S(n) can skip then:

a) If $\{e_1,e_2,e_3,\dots\dots\}$ is the set of slots left empty by S(n) when operating normally, then the set of slots that it leaves empty after having skipped is $\{e_2,e_3,\dots\dots\}$.

b) The transmissions that must be relocated lie all in the interval $[1,e_1-1]$ and are those corresponding to the first message of each node.

c) After skipping, the first message from node 1 is transmitted at the second slot and the first message from node i, $i = 2,\dots,n$, is transmitted at the slot $e_{2(i-1)}$, being $e_{2(i-1)}$ the second slot left empty by the subsystem S(i-1) when operating normally. Therefore $T_i \geq e_{2(i-1)}$ for $i = 2,\dots,n$.

Proof. Let $\{e_1,e_2,e_3,\dots\dots\}$ be the set of slots left empty by S(n) when operating normally. Then the interval $[1,e_1-1]$ is completely filled with transmissions, which are fulfilled in the interval $[2,e_1]$ if the system skips. Since at the slot e_1 the system does not generate any message, it is clear that from this slot on all the subsequent messages will find available in $[e_1+1, +\infty)$ the same slots that they are able to gain when the mechanism behaves well.

Therefore when the system skips it leaves empty the same set of slots as before, except e_1, what proves a), and the transmissions that must be relocated lie all in the interval $[1, e_1 - 1]$.

To finish the proof assume that the system has skipped. It is clear that the only message from the node 1 whose transmission has to be delayed is the first one. As for the node i, $i = 2, \ldots, n$, we restrict our attention to the system S(i-1) consisting of the first i-1 nodes, that clearly is crisis-free and can skip, and regard the situation as this system having skipped. The messages from node i must be transmitted at slots left now empty by S(i-1). The first of these empty slots is $e_{2(i-1)}$, as we have just proved above, and thus the first message from node i is transmitted at $e_{2(i-1)}$ instead of at $e_{1(i-1)}$ as before. Then $T_i \geqq e_{2(i-1)}$, which proves c). From this inequality follows that all the subsequent messages of the node i are generated from the slot $e_{2(i-1)}+1$ on. Since in the interval $[e_{2(i-1)}+1, +\infty)$ the system S(i-1) leaves empty the same slots as before skipping, it is clear that all the messages from the node i, except the first one, can be transmitted at the same slot as earlier. This ends the proof of b).

The posed problem of finding a necessary and sufficient condition for a crisis-free non saturated system S(n) under a FPD to be able to skip is trivial for $n = 1$, since any non saturated system with only one node can skip. For $n \geqq 2$ we have:

THEOREM 3. For $n \geqq 2$, a crisis-free non saturated system S(n) operating under a FPD can skip if and only if $T_i \geqq e_{2(i-1)}$ for $i = 2, \ldots, n$.

Proof. Assume S(n) is a crisis-free non saturated system such that $T_i \geqq e_{2(i-1)}$ for $i = 2, \ldots, n$. We must prove that S(n) can skip.

Suppose that S(n) skips. As S(n) is non saturated $T_1 > 1$, and it is clear that all the messages from the node 1 can be transmitted in time since the only one whose transmission must be delayed is the first message, that can be fulfilled at the second slot.

Reasoning by induction on the nodes, suppose that all the messages from the i-1 first nodes can be transmitted without crisis and let us prove that this is also true for the messages coming from the node i, i > 1.

We restrict our attention to the system S(i-1) which, by the inductive hypothesis, has skipped being able to do it. By Theorem 2, a) the first slot left now empty by S(i-1) is $e_{2(i-1)}$. Since by hypothesis $T_i \geq e_{2(i-1)}$, it follows that the first message from the node i can be fulfilled without crisis at $e_{2(i-1)}$. As for its subsequent messages, they will find still available the same slots as before because S(i-1) leaves all of them empty, again by Theorem 2, a). This proves that S(n) can skip.

The converse has already been proved in Theorem 2, c).

2.2. A characterization of the steady crisis-free systems.

Now we are ready to give a necessary and sufficient condition for a system S(n) to be steady crisis-free. As a system with only one node is obviously steady crisis-free, we consider $n \geqslant 2$.

THEOREM 4. For $n \geq 2$, a system S(n) operating under a FPD is steady crisis-free if and only if it verifies:

1) $\sum_{i=1}^{n-1} 1/T_i < 1.$

2) $T_i \geq e_{2(i-1)}$ for $i = 2,\dots,n-1$ and $T_n \geq e_{1(n-1)}$.

Proof. Suppose S(n) is a system operating under a FPD such that 1) and 2) hold. We shall prove that S(n) is steady crisis-free.

In the first place we claim that S(n) is crisis-free. To see this let us consider the systems S(i) consisting of the first i nodes, i = 1,...,n. S(1) is crisis-free non saturated since 1) implies $T_1 > 1$. As for S(2), since $e_{2(1)} > e_{1(1)}$ and by 2) $T_2 \geq e_{2(1)}$, it follows that $T_2 \geq e_{1(1)}$. Then by R3) S(2) is crisis-free, and from the hypothesis 1) follows that S(2) is non saturated. Repeating the argument for S(3), we obtain that S(3) is crisis-free non saturated. By iteration, we have that S(1), S(2),...,S(n-1) are crisis-free non saturated. Applying again R3) to S(n-1) and $T_n \geq e_{1(n-1)}$ we have that S(n) is crisis-free.

Suppose now that the node k, k ≠ 1, gains the first slot. We have to prove that all the messages from the system can still be transmitted without crisis.

When the mechanism behaves well the first message from the node k is transmitted at the slot $e_{1(k-1)}$ and the interval [1, $e_{1(k-1)}-1$] is completely filled with messages from the k-1 first nodes. As the node k gains the first slot, such messages must be transmitted in the interval [2, $e_{1(k-1)}$]. And this is all the change required because the subsequent messages from the first k nodes as well as all the messages from the remaining n-k will find available in the interval [$e_{1(k-1)}+1$, $+\infty$) the same slots that they are able to gain when the mechanism behaves well.

In order to prove that those messages from the first k-1 nodes find suitable slots in the interval [2, $e_{1(k-1)}$] to be fulfilled in time, we may regard the situation as the system S(k-1) having skipped. And it actually can skip, because if k = 2, S(1) can skip since $T_1 > 1$, and if k ≧ 3, from Theorem 3 follows that S(k-1) can skip since it is crisis-free non saturated and $T_i \geq e_{2(i-1)}$ for i = 2,...,k-1 by hypothesis 2).

This proves that S(n) is steady crisis-free.

Conversely, assume that S(n) is a steady crisis-free system. As a steady crisis-free system is crisis-free, it is clear that S(n-1) is crisis-free non saturated. Thus 1) holds and also the last inequality in 2), which follows from R3).

Suppose now that the node n gains the first slot. Since S(n) is steady crisis-free, this means that S(n-1) can skip, and then the remaining inequalities in 2) follow applying Theorem 3 to S(n-1). This ends the proof of the theorem.

From this theorem easily follows a useful criterion for enlarging a given steady crisis-free system:

THEOREM 5. Let S(n) be a steady crisis-free non saturated system operating under a FPD. Then the system S(n+1) obtained by adding a n+1-th node at the bottom of the stack is steady crisis-free if and only if $T_n \geq e_{2(n-1)}$ and $T_{n+1} \geq e_{1(n)}$.

Proof. Immediate.

Applying Theorems 1 and 5 and property R1) we can conclude that

THEOREM 6. Let S(n) be a system operating under a FPD. Then

I) If $n \leq T_{min}$, S(n) is steady crisis-free.

II) If $n > T_{min}$ then S(n) is steady crisis-free if and only if it verifies:

$$\text{for } i = T_{min},\dots,n-1 \quad \exists\, x_i \in [i+1, T_i] \text{ such that } 2 + \sum_{h=1}^{i-1} \lceil \frac{x_i}{T_h} \rceil = x_i$$

$$\text{and} \quad \exists\, x_n \in [n, T_n] \text{ such that } 1 + \sum_{h=1}^{n-1} \lceil \frac{x_n}{T_h} \rceil = x_n \, .$$

Proof. Assertion I) is Theorem 1.

Suppose $n > T_{min}$ and that S(n) verifies the conditions in II). We must prove that S(n) is steady crisis-free. Let $k = T_{min}$. As S(k) is steady crisis-free, it is crisis-free. By hypothesis there exists $x_k \in [k+1, T_k]$ such that $f_{(k-1)}(x_k) = \sum_{h=1}^{k-1} \lceil \frac{x_k}{T_h} \rceil = x_k - 2$ which implies ([2]) that in the interval $[1, x_k]$ the system S(k-1) generates exactly $x_k - 2$ messages and therefore in this interval there are empty slots, at least two. From $f_{(k-1)}(x_k) = x_k - 2$ follows by R2) that $x_k \geq e_{2(k-1)}$ and therefore $T_k \geq e_{2(k-1)}$ (*).

Since there exists $x_{k+1} \in [k+2, T_{k+1}]$ such that $f_{(k)}(x_{k+1}) = x_{k+1} - 2$, the number of messages generated by S(k) in the interval $[1, x_{k+1}]$ is $x_{k+1} - 2$, so S(k) leaves empty at least two of its slots, $e_{1(k)}$ and $e_{2(k)} \leq x_{k+1}$. As $e_{1(k)} < e_{2(k)} \leq x_{k+1} \leq T_{k+1}$, we have that $T_{k+1} \geq e_{1(k)}$ (**). Applying Theorem 5 to S(k), (*) and (**), it follows that S(k+1) is steady crisis-free. Iterating this argument we obtain that S(n) is steady crisis-free.

Conversely, if S(n) is steady crisis-free then it verifies the inequalities 2) in Theorem 4. On the other hand, $e_{2(i-1)} \geq i+1$ and $e_{1(n-1)} \geq n$ always hold. From this and R2) follows that the inequalities 2) may be written as the conditions in II).

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